

Suppression of superconductivity by Neel-type magnetic fluctuations in the iron pnictides

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Motivated by recent experimental detection of Neel-type $((\pi, \pi))$ magnetic fluctuations in some iron pnictides, we study the impact of competing (π, π) and $(\pi, 0)$ spin fluctuations on the superconductivity of these materials. We show that even short-range, weak Neel fluctuations strongly suppress the s^{+-} state, with the main effect arising from a repulsive contribution to the s^{+-} pairing interaction, complemented by low frequency inelastic scattering. Further increasing the strength of the Neel fluctuations leads to a low- T_c d-wave state, with a possible intermediate $s + id$ phase. The results suggest that the absence of superconductivity in a series of hole-doped pnictides is due to the combination of short-range Neel fluctuations and pair-breaking impurity scattering, and also that T_c of the pnictides could be further increased if residual (π, π) fluctuations could be eliminated.

The proximity of the superconducting state (SC) to a “stripe” spin-density wave instability (SDW) in the phase diagrams of the recently discovered iron-based superconductors [1] (FeSC) prompted the proposal that SDW spin fluctuations provide the pairing mechanism [2]. Indeed, the Fermi surface of many iron pnictides consists of separate electron and hole pockets with the electron pockets displaced from the central hole pockets by the SDW ordering vector $\mathbf{Q}_{\text{SDW}} = (\pi, 0) / (0, \pi)$ (see Fig. 1). In this situation, even weak SDW fluctuations may overcome a strong on-site repulsion giving rise to an s^{+-} SC state, in which the gap function has one sign on the electron pockets and another sign on the hole pockets [3].

As can be seen from Fig. 1, the two electron pockets are connected by the momentum $\mathbf{Q}_{\text{Neel}} = (\pi, \pi)$ suggesting that Neel-type magnetic fluctuations may also be important [4]. These fluctuations favor a d-wave SC state in which the gap function has opposite sign in the two electron pockets. On the theory side, first-principle and Hartree-Fock calculations find that the Neel state is locally stable, but with a higher energy than the SDW state [5, 6], while random phase approximation (RPA) calculations performed in the paramagnetic phase find a peak

in the magnetic susceptibility at \mathbf{Q}_{Neel} , which is however weaker than the peak at \mathbf{Q}_{SDW} [7].

Recent experiments also evidence the importance of Neel order and fluctuations in some FeSCs. Neutron scattering measurements [8] revealed that even at small x , $\text{Ba}(\text{Fe}_{1-x}\text{Mn}_x)_2\text{As}_2$ exhibits spin fluctuations peaked at \mathbf{Q}_{Neel} , in addition to the SDW fluctuations peaked at \mathbf{Q}_{SDW} . NMR measurements [9] confirmed that these Neel fluctuations couple to the conduction electrons. Because the entire family of “in-plane” hole-doped $\text{Ba}(\text{Fe}_{1-x}\text{M}_x)_2\text{As}_2$ compounds ($M = \text{Mn, Cr, Mo}$) [10] displays SDW order at $x = 0$ and Neel order at $x = 1$, we expect that competing Neel and SDW fluctuations will be found across the whole material family, making this series a promising system to tune these competing spin fluctuations. While superconducting states have not been reported in these materials to date [11] (in contrast to the electron-doped counterparts $M = \text{Co, Ni, Rh, Pt, Cu}$, where SC is always observed [12]) higher pressure experiments on samples with lower residual resistivity may reveal SC.

There is also indirect evidence for Neel fluctuations in the extremely electron-doped $\text{A}_y\text{Fe}_{2-x}\text{Se}_2$ compounds [13]. In these materials, the near absence of Fermi surface pockets in the center of the Brillouin zone suggests that SDW fluctuations and the s^{+-} state are disfavored, while the square-like shape of the electron pockets is expected to enhance the (π, π) fluctuations [14]. Chemical substitution on the A site or application of pressure [15], however, can create a small pocket in the center of the Brillouin zone, which could support $(\pi, 0)$ fluctuations and s^{+-} superconductivity.

The effect of competing spin fluctuations on FeSCs is thus of experimental and theoretical interest. In this paper, we address the problem via a multi-band Eliashberg approach [16, 17] in which the spin fluctuations are modeled as bosons with spectral density determined experimentally or chosen phenomenologically, while their effect on the electrons is determined from the one-loop approximation to the self energy (see diagrams in Fig. 1). This

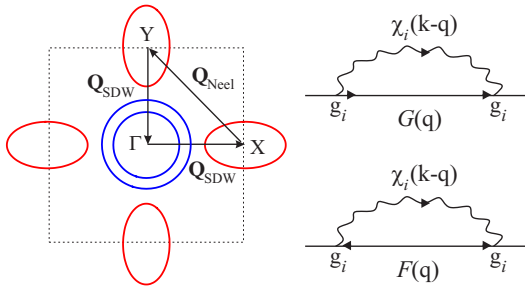


Figure 1: (left panel) Schematic Fermi surface configuration in the 1-Fe Brillouin zone, with two central hole pockets and two electron pockets. (right panel) Self-energy diagrams of the Eliashberg equations: normal component (upper panel) and anomalous component (lower panel).

approximation has been extensively employed in studies of cuprates [18–20], ferromagnetic SC [21], and pnictides [22]. Although some complications arise for two dimensional electrons coupled to two dimensional spin fluctuations, it is generally accepted that the one-loop calculation gives a reasonable representation of the physics when the typical boson frequency is small compared to the Fermi energy [20].

Our calculation goes beyond previous work [22] by incorporating both SDW and Neel fluctuations and including the Coulomb pseudo-potential, as well as by using the experimentally determined spin fluctuation spectrum instead of the single-pole approximation employed previously. We find that the Coulomb pseudo-potential has only a weak effect on the dominant s^{+-} state but that even weak, short-range Neel fluctuations strongly suppress the transition temperature $T_c^{s\text{-wave}}$. If sufficiently strong, the Neel fluctuations may induce a d-wave state, but the transition temperature is found to be much lower than the optimal T_c for the s^{+-} state. The transition between s^{+-} and d -SC may either occur via an intermediate time reversal symmetry-breaking $s + id$ state [23, 24] or, if the impurity scattering is stronger, via an intermediate non-SC state separating the two regions (see Fig. 2). We further show that the low temperature s^{+-} state existing in the presence of combined SDW and Neel fluctuations is easily suppressed by impurities or, presumably, by a competing ordered state, such as the SDW [25, 26].

To gain insight into the results, we use functional derivative methods pioneered by Bergmann and Rainer [27, 28]. We find that the strong suppression of the s^{+-} state comes mostly from a repulsive s^{+-} pairing interaction induced by the Neel fluctuations, although pair-breaking inelastic scattering plays some role. Finally, we discuss the implications of our results not only to the SC of the in-plane hole-doped pnictides, but also to the value of T_c in the FeSCs in general.

Our model consists of a two-dimensional Fermi surface with two hole pockets (Γ , density of states N_Γ) at the center of the Brillouin zone and two electron pockets (X and Y , density of states N_X) displaced from the center by the momenta $(\pi, 0)$ and $(0, \pi)$ (Fig. 1) [29]. For simplicity, hereafter we assume that these two hole pockets are degenerate - our results do not depend on this simplification. Following Ref. [30], we set $r = N_X/N_\Gamma = 0.65$. The electrons are coupled to two types of low-energy bosonic excitations, namely, SDW spin fluctuations peaked at $(\pi, 0) / (0, \pi)$ and Neel spin fluctuations peaked at (π, π) . Experiment (Refs.[8, 31]) indicates that in the paramagnetic phase these excitations are described by diffusive dynamic susceptibilities:

$$\chi_i^{-1}(\mathbf{Q}_i + \mathbf{q}, \Omega_n) = \xi_i^{-2} + q^2 + \gamma_i^{-1} |\Omega_n| \quad (1)$$

Here, \mathbf{q} is the momentum deviation from the ordering vector \mathbf{Q}_i (all lengths are in units of the lattice parameter a) and Ω_n is the bosonic Matsubara frequency. The quantity that actually enters the Eliashberg equations

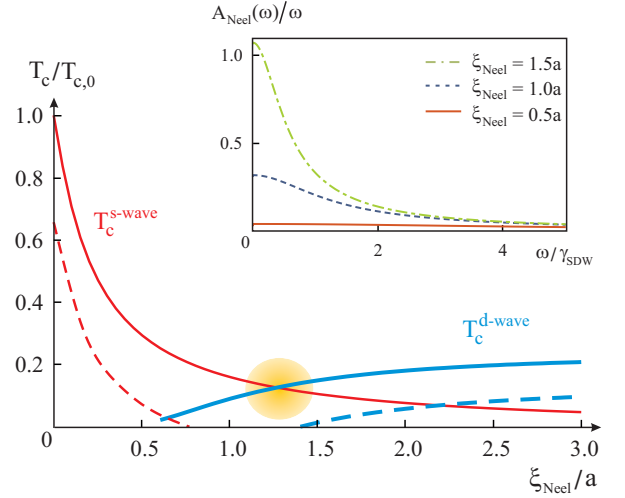


Figure 2: Transition temperatures T_c of the s-wave (red/light curve) and d-wave (blue/heavy curve) states as function of the Neel magnetic correlation length ξ_{Neel} , for $r = 0.65$, $\gamma_{\text{Neel}}/\gamma_{\text{SDW}} = 0.33$, $\lambda_{\text{Neel}}/\lambda_{\text{SDW}} = 2$, and $\mu^* = 0.8$. $T_{c,0} \approx 0.1\gamma_{\text{SDW}}$ is the s^{+-} transition temperature for $\xi_{\text{Neel}} = 0$. The shaded area denotes the regime where the two states have similar transition temperatures and a possible $s + id$ state may occur. The dashed lines show the behavior of the system in the presence of impurity scattering. The inset shows the frequency dependence of the spectral function $A_{\text{Neel}}(\omega)$ of the Neel fluctuations for different values of ξ_{Neel} .

is the spectral function integrated over the momentum component q_{\parallel} parallel to the Fermi surface and evaluated at $q_{\perp} = 0$, i.e. $A_{\text{Neel}}(\omega) = \int dq_{\parallel} \text{Im} \chi_{\text{Neel}}(q_{\parallel}, \omega)$. This spectral function gives rise to the Matsubara-axis interaction $a^{(i)}(\Omega_n) = \xi_i / \sqrt{1 + |\Omega_n| \gamma_i^{-1} \xi_i^2}$ which enters the Eliashberg equations as described below. The spin fluctuations in each momentum channel i are described by two parameters: the Landau damping γ_i , which sets the energy scale of each fluctuation spectrum, and the correlation length ξ_i , which sets both the strength and the spatial/temporal correlations of the spin fluctuations. We will tune the spectrum by varying ξ_i .

Because the Landau damping originates from the low-energy decay of the spin excitations into electron-hole pairs, γ_i is determined by the electron-boson coupling constant g_i and the densities of states. Within a standard spin fluctuation model there is one adjustable parameter, which we choose to be g_{SDW} and which we set to yield $T_{c,0}^{s\text{-wave}} \approx 30$ K. The remaining quantities are determined from the experimentally measured γ_i and the ratio of densities of states r (see the Supplementary Material for details). Following the experimental results of Ref. [8], we use $\gamma_{\text{Neel}}/\gamma_{\text{SDW}} \approx 0.33$ with $\gamma_{\text{SDW}} \approx 25$ meV; the value of g_{Neel} follows from the relationship between $\gamma_{\text{Neel}}/\gamma_{\text{SDW}}$ and $g_{\text{Neel}}/g_{\text{SDW}}$. Finally, we set $\xi_{\text{SDW}} = 5a$ throughout our calculations, varying the correlation length of the Neel fluctuations ξ_{Neel} . Our results do not change significantly for smaller values of ξ_{SDW} .

To obtain the transition temperatures in the s and d -wave channels we linearize the Eliashberg equations in the superconducting quantities and solve the resulting equations for the anomalous component $W_{\alpha,n}$ and the normal component $Z_{\alpha,n} = \text{Im}\Sigma_{\alpha,n}^N/i\omega_n$ of the self-energy (the real part of Σ^N just renormalizes the band dispersions, possibly differently for different pockets [16, 17, 32]). These quantities are averaged over each Fermi pocket becoming functions only of the Fermi pocket label α and the fermionic Matsubara frequency $\omega_n = (2n+1)\pi T$. With the aid of the auxiliary “gap functions” $\bar{\Delta}_{\Gamma,n} \equiv \frac{W_{\Gamma,n}}{Z_{\Gamma,n}|\omega_n|\sqrt{N_X}}$ and $\bar{\Delta}_{(X/Y),n} \equiv \frac{W_{(X/Y),n}}{Z_{X,n}|\omega_n|\sqrt{N_{\Gamma}}}$, the linearized gap equation is expressed as a matrix equation in Matsubara (indices n, m) and band (indices α, β) spaces $\sum_{m,\beta} K_{nm}^{\alpha\beta} \bar{\Delta}_{\beta,m} = 0$, with the kernel:

$$(K_{nm}^{\alpha\beta}) = - \begin{pmatrix} \delta_{nm} \frac{Z_{\Gamma,n}|\omega_n|}{T} & \frac{\lambda_{\text{SDW}}}{2} a_{nm}^{(1)} & \frac{\lambda_{\text{SDW}}}{2} a_{nm}^{(1)} \\ \lambda_{\text{SDW}} a_{nm}^{(1)} & \delta_{nm} \frac{Z_{X,n}|\omega_n|}{T} & \lambda_{\text{Neel}} a_{nm}^{(2)} \\ \lambda_{\text{SDW}} a_{nm}^{(1)} & \lambda_{\text{Neel}} a_{nm}^{(2)} & \delta_{nm} \frac{Z_{X,n}|\omega_n|}{T} \end{pmatrix} - \frac{1}{2} \left[\frac{\mu^*}{1+r} - \frac{\tau^{-1}}{T} \frac{\delta_{nm}}{r} \right] \begin{pmatrix} 2 & \sqrt{r} & \sqrt{r} \\ 2\sqrt{r} & r & r \\ 2\sqrt{r} & r & r \end{pmatrix} \quad (2)$$

Here we have introduced the matrix elements coming from the bosonic modes $a_{nm}^{(i)} = a^i(\omega_n - \omega_m)$ ($i = 1$ corresponds to SDW and $i = 2$, to Neel fluctuations) and the dimensionless coupling constants $\lambda_{\text{SDW}} = 2g_{\text{SDW}}^2\sqrt{N_{\Gamma}N_X}$, $\lambda_{\text{Neel}} = g_{\text{Neel}}^2N_X$. T is the temperature. We also introduce an upper frequency cutoff $\Lambda = 8\gamma_{\text{SDW}}$, corresponding to the energy scale of the bottom/top of the electron/hole bands, and we assume that μ^* is a bare Coulomb interaction renormalized in the standard way by higher energy processes. τ^{-1} is the scattering rate associated with non-magnetic point impurities and the $Z_{\alpha,n}$ functions are obtained analytically (see Supplementary Material). The Coulomb pseudo-potential favors solutions with $\sum_{\alpha} N_{\alpha} \Delta_{\alpha} = 0$.

Reflecting the tetragonal symmetry of the system, this matrix equation supports two different types of solution: the s -wave state $\bar{\Delta}_{X,n} = \bar{\Delta}_{Y,n}$, with either s^{++} ($\bar{\Delta}_{\Gamma,n} \propto \bar{\Delta}_{X,n}$) or s^{+-} ($\bar{\Delta}_{\Gamma,n} \propto -\bar{\Delta}_{X,n}$) structure, and the d -wave state $\bar{\Delta}_{X,n} = -\bar{\Delta}_{Y,n}$. The solution in a given symmetry channel is obtained when the largest eigenvalue η of the matrix (2) vanishes. Since our calculations never yield an s^{++} state, hereafter we use the terms s -wave and s^{+-} to refer to the same state. Due to limitations of the size of the matrices that can be diagonalized, and since the matrix size scales as Λ/T , we resolve $T_c \gtrsim 10^{-3}\gamma_{\text{SDW}}$. Hereafter, we set $\lambda_{\text{SDW}} = 0.4$ and the Coulomb pseudo-potential $\mu^* = 0.8$, which gives, in the absence of competing Neel fluctuations, $T_{c,0}^{\text{s-wave}} \approx 30\text{K}$ and implies $\lambda_{\text{Neel}} = 0.8$.

Fig. 2 shows our principal results: the dependence of the SC transition temperature T_c on the strength of Neel fluctuations (parametrized by the Neel correlation length ξ_{Neel}). The light solid line (red online) shows the transition temperature $T_c^{\text{s-wave}}$ for the s^{+-} channel in the

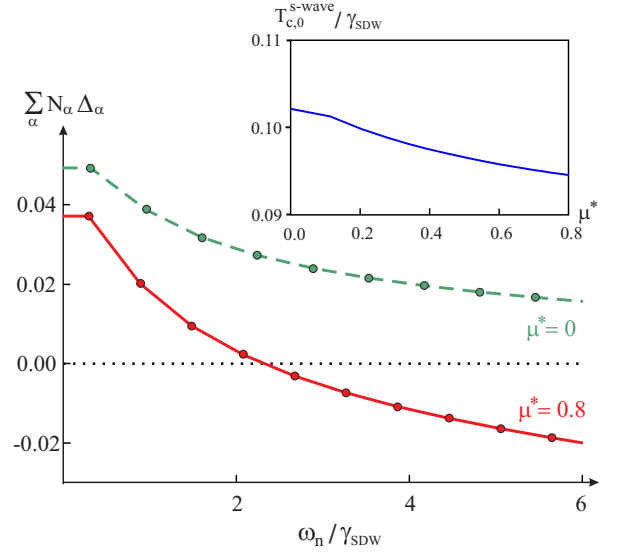


Figure 3: Averaged s^{+-} gap function $\sum_{\alpha} N_{\alpha} \Delta_{\alpha}$ across the different pockets at $T_{c,0}^{\text{s-wave}}$ as function of Matsubara frequency ω_n (in units of γ_{SDW}), for $\mu^* = 0$ (green/dashed curve) and $\mu^* = 0.8$ (red/solid curve). The inset shows $T_{c,0}^{\text{s-wave}}$ (in units of γ_{SDW}) as function of μ^* . Although here we used $\xi_{\text{Neel}} = 0$, a similar behavior holds for $\xi_{\text{Neel}} \neq 0$.

absence of impurity scattering. One sees that even weak, short-range fluctuations strongly suppress s^{+-} SC, but that once $T_c^{\text{s-wave}}$ has been substantially reduced, the additional suppression effect caused by further increasing ξ_{Neel} is small. Sufficiently strong Neel correlations produce a d -wave solution (heavier solid line, blue online) with $T_c^{\text{d-wave}}$ that eventually becomes larger than $T_c^{\text{s-wave}}$ but always remains small compared to the maximum $T_c^{\text{s-wave}}$. In our linearized theory the transition between s -wave and d -wave superconductors appears as a discontinuous change in the nature of the state, but the considerations of [23] suggest that the nonlinear terms not included here change the physics so that the s and d states are separated by a range of $s + id$ superconductivity (shaded area). The dashed lines show the behavior in the presence of impurity scattering, which is pair-breaking for both s^{+-} and d -wave superconductivity. Sufficiently strong impurity scattering can disconnect the two SC states, leaving an intermediate non-SC regime.

We also analyze the impact of the Coulomb pseudo-potential μ^* on the s^{+-} state - the d -wave state avoids the Coulomb repulsion. Fig. 3 shows the pocket-averaged s^{+-} gap function $\sum_{\alpha} N_{\alpha} \Delta_{\alpha}$ both in the presence and in the absence of μ^* . To avoid the local repulsion, the averaged order parameter changes sign at a non-zero Matsubara frequency, although the sign of each individual gap does not necessarily change. This is the multi-band analogue of the response of a single-band s -wave superconductor to the local repulsion. For all values of ξ_{Neel} we have studied, neither $\Delta_{n=0}$ nor $T_c^{\text{s-wave}}$ (shown in the inset) are substantially altered by μ^* , in agreement with

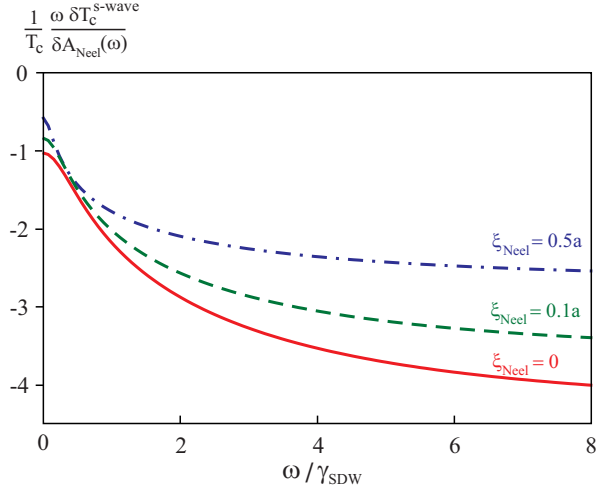


Figure 4: Functional derivative $T_c^{-1} \omega \delta T_c^{s\text{-wave}} / \delta A_{\text{Neel}}(\omega)$ as function of frequency ω (in units of γ_{SDW}) for the cases $\xi_{\text{Neel}} = 0$ (red/solid line), $\xi_{\text{Neel}} = 0.1a$ (green/dashed line), and $\xi_{\text{Neel}} = 0.5a$ (blue/dotted dashed line).

the weak-coupling analysis of Ref. [33].

We now turn to the physics of the decrease of $T_c^{s\text{-wave}}$ caused by Neel fluctuations. Increasing ξ_{Neel} increases the spin fluctuation intensity and changes its functional form (see the inset of Fig. 2). To understand in more detail the effects on s^{+-} pairing, we use the functional derivative techniques introduced by Bergmann and Rainer [27, 28] to analyze how different frequency regions of the Neel spectral function affect $T_c^{s\text{-wave}}$. In Fig. 4 we plot the functional derivative

$$\frac{1}{T_c} \frac{\omega \delta T_c}{\delta A_{\text{Neel}}(\omega)} = \left[\hat{\Delta} \frac{\omega \delta \hat{K}}{T_c \delta A_{\text{Neel}}(\omega)} \hat{\Delta} \right] / \left(-\frac{\partial \eta}{\partial T_c} \right)_{\eta=0} \quad (3)$$

for different values of ξ_{Neel} . Previous work [27, 28] has shown that the low frequency regime captures the pair-breaking effects of inelastic scattering while the high frequency regime expresses the changes to the pairing interaction. The larger magnitude of $\delta \log T_c / \delta A_{\text{Neel}}$ at high frequencies shows that in the pnictides the dominant effect of the Neel fluctuations is to provide a negative contribution to the s^{+-} pairing interaction, with the extra pair-breaking effect of the induced low frequency inelastic scattering being non-zero but less important. Because Fig. 4 shows that the logarithmic derivative of T_c is a slow function of ξ_{Neel} we conclude that the initial steep drop and subsequent flattening of the T_c curve shown in Fig. 2 is due in large part to the variation of T_c itself. Additionally, as ξ_{Neel} is increased the Neel fluctuation spectrum shifts to lower frequencies (see the inset of Fig. 2), where the pair-breaking is less effective. However, additional physics is also at play. Consider, for instance, the weak-coupling limit of two effective competing pairing interactions λ_s and λ_d . We obtain

$$T_c \propto \exp \left[-1 / \left(\sqrt{\lambda_s^2 + \lambda_d^2} - \lambda_d \right) \right] \text{ and}$$

$$\frac{d \log T_c}{d \lambda_d} \propto - \frac{1}{\left(\lambda_s^2 + \lambda_d^2 - \lambda_d \sqrt{\lambda_s^2 + \lambda_d^2} \right)} \quad (4)$$

which is larger in magnitude for larger λ_d , implying an opposite ordering of the curves to that seen in Fig. 4. The difference between our results and this simple weak-coupling estimate arises because the gap function self-consistently adjusts to the pairing potential, so that for larger ξ_{Neel} the gap function decreases more rapidly with frequency, thereby minimizing the depairing effects of the Neel fluctuations (see Supplementary Material).

Our results offer a possible explanation for the puzzling behavior of the hole-doped $\text{Ba}(\text{Fe}_{1-x}\text{M}_x)_2\text{As}_2$ series ($M = \text{Mn, Cr, Mo}$) [10], which, in contrast to its electron-doped counterpart ($M = \text{Co, Ni, Rh, Pt, Cu}$) [12], does not display SC. The short-range Neel fluctuations induced by the dopants, which were observed experimentally for low concentrations of $M = \text{Mn}$ [8, 9], suppress the s^{+-} state without giving rise to a high-temperature d-wave state. This low- T_c s^{+-} state, in turn, can be easily suppressed, for example by impurity scattering or by another competing ordered state, such as the SDW state [25] observed at low x . We suggest that improving the purity of the samples and applying pressure to suppress the SDW state may reveal either a weakened s^{+-} state or perhaps a low T_c d-wave state. Similarly, in the extremely electron-doped $A_y\text{Fe}_{2-x}\text{Se}_2$ systems [13, 34, 35], where for small y the hole pocket is generally absent and d-wave superconductivity is discussed, adding holes by changing A [14] or by applying pressure [15] should produce the reverse competition.

More generally, we expect that most of the FeSC families will display Neel-type fluctuations due to the existence of two matching electron pockets separated by \mathbf{Q}_{Neel} . Our findings show that, even when they are weak, these (π, π) fluctuations will strongly suppress $T_c^{s\text{-wave}}$. This suggests that one possible route to optimizing T_c is to minimize these fluctuations, for example by making the two electron pockets unequal via a tetragonal symmetry breaking [36]. Interestingly, torque magnetometry measurements found such a tetragonal symmetry breaking above T_c in some optimally doped FeSCs [37]. In Ref. [38], it was also observed that a small strain applied along the orthorhombic axis can enhance T_c .

In summary, our results open a new route to explore unconventional superconductivity in multi-band systems by controlling competing spin fluctuations. In particular, Neel fluctuations have a strong effect on the s^{+-} state of the FeSCs, rapidly reducing T_c and potentially driving a transition from s-wave to d-wave SC (Fig. 2). Depending on the strength of impurity scattering, more exotic states can emerge, such as the $s + id$ state [23]. Note that an $s + id$ state was also suggested to emerge due to other mechanisms, such as extreme doping and Fe-As hybridization [24]. Notice also that, in our model, since there are two SC instabilities in different symme-

try channels, the lower T_c solution, while not present in the ground state, will give rise to a collective excitation which can in principle be detected by Raman scattering [39].

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Supplementary material for “Suppression of superconductivity by Neel-type magnetic fluctuations in the iron pnictides”

I. FORMULATION AND SOLUTION OF ELIASHBERG EQUATIONS

A. Formulation of Equations

The low-energy action describing the coupling between the electrons and the SDW and Neel fluctuations is conveniently expressed in terms of the Nambu operator $\Psi_{\mathbf{k}}^{\dagger} = \left(c_{\Gamma, \mathbf{k}\uparrow}^{\dagger} \ c_{\Gamma, -\mathbf{k}\downarrow} \ c_{\Gamma', \mathbf{k}\uparrow}^{\dagger} \ c_{\Gamma', -\mathbf{k}\downarrow} \ c_{X, \mathbf{k}\uparrow}^{\dagger} \ c_{X, -\mathbf{k}\downarrow} \ c_{Y, \mathbf{k}\uparrow}^{\dagger} \ c_{Y, -\mathbf{k}\downarrow} \right)$, where $c_{\Gamma, \mathbf{k}\sigma}$, $c_{\Gamma', \mathbf{k}\sigma}$, $c_{X, \mathbf{k}\sigma}$, and $c_{Y, \mathbf{k}\sigma}$ correspond to operators on the Γ and Γ' hole pockets, on the X electron pocket at $(\pi, 0)$, and on the Y electron pocket at $(0, \pi)$, respectively. We have:

$$S = \int_k \Psi_{\mathbf{k}}^{\dagger} (\hat{\varepsilon}_{\mathbf{k}} - i\omega_n \hat{1}) \Psi_{\mathbf{k}} + \sum_{i=1}^5 \int_k \chi_i^{-1}(k) \Phi_{i, \mathbf{k}} \cdot \Phi_{i, -\mathbf{k}} + \sum_{i=1}^5 g_i \int_{k, q} \Phi_{i, -\mathbf{k}-\mathbf{q}} \cdot \left(\Psi_{\mathbf{k}}^{\dagger} \hat{\rho}_i \Psi_{\mathbf{q}} \right) \quad (\text{S1})$$

where $k = (\mathbf{k}, \omega_n)$ refers to both momentum and fermionic Matsubara frequency, $\Phi_{i, \mathbf{k}}$ denotes the collective bosonic fields associated with the SDW ($i = 1, \dots, 4$) and Neel ($i = 5$) fluctuations, and $\chi_i(k)$ refers to the corresponding dynamic magnetic susceptibilities. Our indices are defined such that $\chi_{(\Gamma, \Gamma')}(\pi, 0)$ corresponds to $i = 1, 2$, $\chi_{(\Gamma, \Gamma')}(0, \pi)$ to $i = 3, 4$, and $\chi(\pi, \pi)$ to $i = 5$. The coupling constants satisfy $g_{i=1\dots 4} = g_{\text{SDW}}$ and $g_5 = g_{\text{Neel}}$. We also have the band dispersions $\hat{\varepsilon}_{\mathbf{k}} = \text{diag}_4(\varepsilon_{\Gamma, \mathbf{k}}, \varepsilon_{\Gamma', \mathbf{k}}, \varepsilon_{X, \mathbf{k}}, \varepsilon_{Y, \mathbf{k}}) \otimes \tau_3$, where τ_i are Pauli matrices in Nambu space. For a spin-rotationally invariant system, and for the case of singlet pairing, we can focus on the z -axis projection of $\hat{\rho}_i$ [1], given by:

$$\hat{\rho}_1 = \begin{pmatrix} 0 & 0 & \tau_0 & 0 \\ 0 & 0 & 0 & 0 \\ \tau_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \hat{\rho}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_0 & 0 \\ 0 & \tau_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \hat{\rho}_3 = \begin{pmatrix} 0 & 0 & 0 & \tau_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \tau_0 & 0 & 0 & 0 \end{pmatrix}$$

$$\hat{\rho}_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_0 \\ 0 & 0 & 0 & 0 \\ 0 & \tau_0 & 0 & 0 \end{pmatrix}; \hat{\rho}_5 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_0 \\ 0 & 0 & \tau_0 & 0 \end{pmatrix} \quad (\text{S2})$$

The Eliashberg equations are obtained by calculating the one-loop self-energy

$$\hat{\Sigma}_k = \sum_i g_i^2 \int_q \chi_i(k-q) \hat{\rho}_i \hat{G}_q \hat{\rho}_i \quad (\text{S3})$$

with $\hat{G}_k^{-1} = \hat{G}_{0,k}^{-1} - \hat{\Sigma}_k$ and $\hat{G}_{0,k}^{-1} \equiv i\omega_n \hat{1} - \hat{\epsilon}_k$.

B. Reformulation of Equations

To solve the self-consistent system of equations, Eq. S3, we rearrange them into a form more convenient for numerical solution. We write the self energy as

$$\hat{\Sigma}_k = i\omega_n (\hat{1} - \hat{Z}_k) \otimes \tau_0 + \hat{W}_k \otimes \tau_1 + \hat{\zeta}_k \otimes \tau_3 \quad (\text{S4})$$

where $\hat{Z}_k = \text{diag}_4(Z_{\Gamma,k}, Z_{\Gamma',k}, Z_{X,k}, Z_{Y,k})$ and $\hat{\zeta}_k = \text{diag}_4(\zeta_{\Gamma,k}, \zeta_{\Gamma',k}, \zeta_{X,k}, \zeta_{Y,k})$ are the imaginary and real parts of the normal component, respectively, and $\hat{W}_k = \text{diag}_4(W_{\Gamma,k}, W_{\Gamma',k}, W_{X,k}, W_{Y,k})$ is the anomalous component of the self-energy. As we discussed in the main text, the real part $\zeta_{\alpha,k}$ renormalizes each α band dispersion (possibly in different ways [2]), and will not be discussed here.

Hereafter we will consider two degenerate hole pockets ($N_{\Gamma} = N_{\Gamma'}$ and $\chi_{\Gamma}(\mathbf{Q}_i) = \chi_{\Gamma'}(\mathbf{Q}_i)$), implying $W_{\Gamma} = W_{\Gamma'}$ and $Z_{\Gamma} = Z_{\Gamma'}$. Using the tetragonal symmetry of the system, we have $\chi(0, \pi) = \chi(\pi, 0)$ and $Z_X = Z_Y$, reducing the number of self-consistent equations to five. We integrate over the momentum component q_{\perp} perpendicular to the Fermi surface, using the fact that the electronic propagator is more sharply peaked at the Fermi level than the bosonic propagator [3]. Next, we linearize the equations by keeping the leading terms in order $W_{\alpha,n}$ [4] and average the gaps along each Fermi pocket. We also include the impurity scattering and the Coulomb pseudo-potential in the standard way, obtaining, for the normal part:

$$\begin{aligned} (Z_{\Gamma,n} - 1)\omega_n &= 2g_{\text{SDW}}^2 N_X T \sum_m \text{sgn}(2m+1) \int dq_{\parallel} \chi_{\text{SDW}}(q_{\parallel}, \omega_n - \omega_m) \\ &\quad + g_u^2 \omega_n \sum_{\alpha} N_{\alpha} + u_{\text{imp}}^2 \text{sgn}(\omega_n) \sum_{\alpha} N_{\alpha} \\ (Z_{X,n} - 1)\omega_n &= T \sum_m \text{sgn}(2m+1) \left[2g_{\text{SDW}}^2 N_{\Gamma} \int dq_{\parallel} \chi_{\text{SDW}}(q_{\parallel}, \omega_n - \omega_m) + g_{\text{Neel}}^2 N_X \int dq_{\parallel} \chi_{\text{Neel}}(q_{\parallel}, \omega_n - \omega_m) \right] \\ &\quad + g_u^2 \omega_n \sum_{\alpha} N_{\alpha} + u_{\text{imp}}^2 \text{sgn}(\omega_n) \sum_{\alpha} N_{\alpha} \end{aligned} \quad (\text{S5})$$

and for the anomalous part:

$$\begin{aligned} W_{\Gamma,n} &= -g_{\text{SDW}}^2 N_X T \sum_m \frac{(W_{X,m} + W_{Y,m})}{Z_{X,m} |\omega_m|} \int dq_{\parallel} \chi_{\text{SDW}}(q_{\parallel}, \omega_n - \omega_m) \\ &\quad - g_u^2 T \sum_m \sum_{\alpha} N_{\alpha} \frac{W_{\alpha,m}}{Z_{\alpha,m} |\omega_m|} + u_{\text{imp}}^2 \sum_{\alpha} N_{\alpha} \frac{W_{\alpha,n}}{Z_{\alpha,n} |\omega_n|} \\ W_{(X/Y),n} &= -2g_{\text{SDW}}^2 N_{\Gamma} T \sum_m \frac{W_{\Gamma,m}}{Z_{\Gamma,m} |\omega_m|} \int dq_{\parallel} \chi_{\text{SDW}}(q_{\parallel}, \omega_n - \omega_m) - g_{\text{Neel}}^2 N_X T \sum_m \frac{W_{(Y/X),m}}{Z_{X,m} |\omega_m|} \int dq_{\parallel} \chi_{\text{Neel}}(q_{\parallel}, \omega_n - \omega_m) \\ &\quad - g_u^2 T \sum_m \sum_{\alpha} N_{\alpha} \frac{W_{\alpha,m}}{Z_{\alpha,m} |\omega_m|} + u_{\text{imp}}^2 \sum_{\alpha} N_{\alpha} \frac{W_{\alpha,n}}{Z_{\alpha,n} |\omega_n|} \end{aligned} \quad (\text{S6})$$

Here, g_u is the coupling to the Coulomb repulsion and u_{imp}^2 is the averaged local impurity potential. The Coulomb repulsion renormalizes all bare interactions, which become $g_i^2 \rightarrow g_i^2 / (1 + g_u^2 N)$, where $N = 2N_{\Gamma} + 2N_X$ is the total density of states.

Using the diffusive expression for the spin susceptibility, Eq. (1) of the main text, yields:

$$\int dq_{\parallel} \chi(q_{\parallel}, \Omega_n) \equiv \int_{-\infty}^{\infty} dq_{\parallel} \frac{1}{|\Omega_n| \gamma_i^{-1} + q_{\parallel}^2 + \xi_i^{-2}} = \pi \xi_i \left(\frac{1}{\sqrt{1 + |\Omega_n| \gamma_i^{-1} \xi_i^2}} \right) \quad (S7)$$

For convenience, we absorb the coefficient π into the coupling constants g_i^2 and introduce the density of states ratio $r = N_X/N_{\Gamma}$, defining the Coulomb pseudo-potential $\mu^* = g_u^2 N / (1 + g_u^2 N)$, the scattering rate $\tau^{-1} = 2N_X u_{\text{imp}}^2$, and the coupling constants $\lambda_{\text{SDW}} = 2g_{\text{SDW}}^2 \sqrt{N_{\Gamma} N_X}$ and $\lambda_{\text{Neel}} = g_{\text{Neel}}^2 N_X$. An important note about the form of the magnetic susceptibility: while its static part comes from high-energy modes not considered in our model, the Landau damping is a direct result of the coupling between the paramagnons and the fermions, as given by Eq. (S1). Thus, the parameter γ_i contains information about the coupling constant g_i . By evaluating the bosonic self-energy to one-loop, we obtain $\gamma_{\text{SDW}}^{-1} = \kappa_{\text{SDW}}^{-1} g_{\text{SDW}}^2 N_X N_{\Gamma}$ and $\gamma_{\text{Neel}}^{-1} = \kappa_{\text{Neel}}^{-1} g_{\text{Neel}}^2 N_X^2$, where κ_i are dimensionless parameters presumably of similar orders of magnitude. This puts constraints on the ratio between the effective couplings $\lambda_{\text{SDW}} = 2g_{\text{SDW}}^2 \sqrt{N_{\Gamma} N_X}$ and $\lambda_{\text{Neel}} = g_{\text{Neel}}^2 N_X$, which is expressed as $\frac{\lambda_{\text{Neel}}}{\lambda_{\text{SDW}}} = \frac{\kappa_{\text{Neel}}}{2\kappa_{\text{SDW}}} \frac{\gamma_{\text{SDW}}}{\gamma_{\text{Neel}}} \sqrt{\frac{N_{\Gamma}}{N_X}}$.

C. Solution of Equations

Substituting the definitions given above into Eq. (S5) and evaluating the sums over Matsubara frequencies yields

$$\begin{aligned} \frac{Z_{\Gamma,n}\omega_n}{T} &= (2n+1) + \sqrt{r} \lambda_{\text{SDW}} \xi_{\text{SDW}} S_{\text{SDW},n} + \frac{\tau^{-1}}{T} \text{sgn}(\omega_n) \left(1 + \frac{1}{r}\right) \\ \frac{Z_{X,n}\omega_n}{T} &= (2n+1) + \left(\frac{\lambda_{\text{SDW}} \xi_{\text{SDW}} S_{\text{SDW},n}}{\sqrt{r}} + \lambda_{\text{Neel}} \xi_{\text{Neel}} S_{\text{Neel},n} \right) + \frac{\tau^{-1}}{T} \text{sgn}(\omega_n) \left(1 + \frac{1}{r}\right) \end{aligned} \quad (S8)$$

with the auxiliary functions:

$$\begin{aligned} S_{i,n} &= \frac{2 \text{sgn}(n)}{\sqrt{\frac{2\pi T \xi_i^2}{\gamma_i}}} \left[\text{Hw} \left(\frac{1}{2}, 1 + \frac{\gamma_i}{2\pi T \xi_i^2} \right) - \text{Hw} \left(\frac{1}{2}, |n| + \frac{\text{sgn}(n) + 1}{2} + \frac{\gamma_i}{2\pi T \xi_i^2} \right) \right] + \text{sgn}(n), \quad n \neq 0, -1 \\ S_{i,n} &= 2 \text{sgn}(n) + 1, \quad n = 0, -1 \end{aligned} \quad (S9)$$

Here, $\text{Hw}(\frac{1}{2}, x)$ is the Hurwitz zeta function for which efficient numerical evaluations exist.

After defining the auxiliary “gap functions” $\bar{\Delta}_{\Gamma,n} \equiv W_{\Gamma,n} / (\sqrt{N_X} Z_{\Gamma,n} |\omega_n|)$ and $\bar{\Delta}_{(X/Y),n} \equiv W_{(X/Y),n} / (\sqrt{N_{\Gamma}} Z_{X,n} |\omega_n|)$ and using the solutions for Z given above, it is straightforward to write down the gap equations (S6) as a matrix equation in Matsubara and band spaces, yielding Eq. (2) of the main text. For numerical computations, it is convenient to use the tetragonal symmetry of the system and split the matrix equation in two: one for the s-wave case $\bar{\Delta}_{X,n} = \bar{\Delta}_{Y,n}$ and another one for the d-wave case $\bar{\Delta}_{X,n} = -\bar{\Delta}_{Y,n}$, yielding two different kernels \hat{K}_s (2×2 matrix) and \hat{K}_d (1×1 matrix). We use standard routines to obtain the leading eigenvalue and corresponding eigenvector of the matrix; the transition temperature is the temperature at which the leading eigenvalue crosses zero.

II. FORM OF THE REAL AXIS SPECTRAL FUNCTION

The spectral function for Neel fluctuations that enters the Eliashberg equations is

$$A_{\text{Neel}}(\omega) = \int dq_{\parallel} \text{Im} \chi_{\text{Neel}}(q_{\parallel}, \omega) \quad (S10)$$

Evaluating the integrals gives

$$A_{\text{Neel}}(\omega) = \frac{\sqrt{2} \xi_{\text{Neel}}}{\pi \sqrt{\gamma_{\text{Neel}}}} \sqrt{\frac{\gamma_{\text{Neel}}^2 + \omega^2 \xi_{\text{Neel}}^4 - \gamma_{\text{Neel}}}{\gamma_{\text{Neel}}^2 + \omega^2 \xi_{\text{Neel}}^4}} \quad (S11)$$

This is plotted for different ξ_{Neel} in inset of Fig. 2 of the main text.

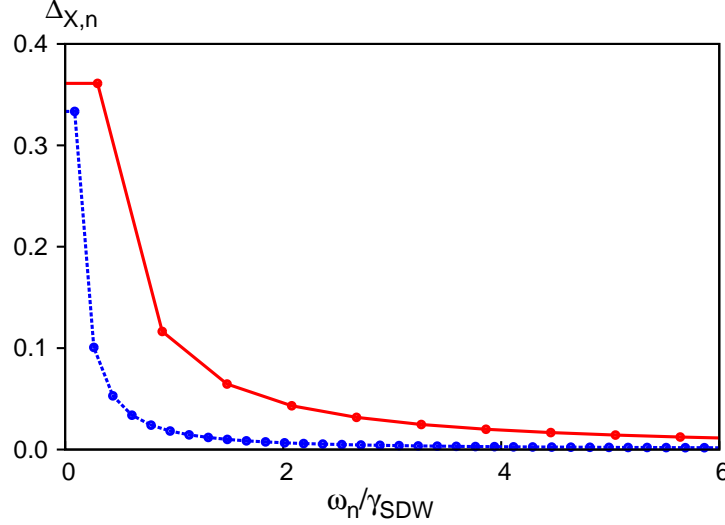


Figure S1: Gap function of the electron pocket $\bar{\Delta}_{X,n}$ as function of Matsubara frequency ω_n (in units of γ_{SDW}) for the parameters used in the main text and $\xi_{\text{Neel}} = 0$ (red/solid line) and $\xi_{\text{Neel}} = 0.5a$ (blue/dashed line).

III. BERGMANN-RAINER ANALYSIS

We here recall how to obtain the functional derivative $\delta T_c^{\text{s-wave}} / \delta A_{\text{Neel}}(\omega)$ via the Bergmann-Rainer approach [5]. The linearized gap equation for the s-wave channel can be cast as an eigenvalue problem

$$\eta \hat{\Delta} = \hat{K}_s \hat{\Delta} \quad (\text{S12})$$

with \hat{K}_s is a matrix in Matsubara and band space and η the largest eigenvalue of \hat{K} . T_c is the temperature at which $\eta = 0$.

Changing $A_{\text{Neel}}(\omega) \rightarrow A_{\text{Neel}}(\omega) + \delta A_{\text{Neel}}(\omega)$ and using the Hellman-Feynman theorem gives

$$\frac{\delta \eta}{\delta A_{\text{Neel}}(\omega)} = \left(\hat{\Delta} \right)^\dagger \frac{\delta \hat{K}_s}{\delta A_{\text{Neel}}(\omega)} \hat{\Delta} \quad (\text{S13})$$

Using the result:

$$\xi_i a_{nm}^{(i)} = \int_0^\infty d\omega \frac{\omega A_i(\omega)}{\omega^2 + |\omega_n - \omega_m|^2} \quad (\text{S14})$$

we obtain:

$$\frac{\delta (K_{nm}^{\alpha\beta})_s}{\delta A_{\text{Neel}}(\omega)} = -\frac{\lambda_{\text{Neel}}}{\omega} \begin{pmatrix} 0 & 0 \\ 0 & \delta_{nm} \text{sgn}(\omega_n) f_n \left(\frac{\omega}{2\pi T} \right) + \frac{\omega^2}{\omega^2 + |\omega_n - \omega_m|^2} \end{pmatrix} \quad (\text{S15})$$

where $f_n(x) = x \text{Im} [\psi(-n + ix) - \psi(n + 1 + ix)]$ and $\psi(x)$ is the digamma function.

Using the fact that η varies smoothly with temperature and rearranging the equation gives

$$\frac{\omega \delta T_c}{T_c \delta A_{\text{Neel}}(\omega)} = \left[\hat{\Delta} \frac{\omega \delta \hat{K}_s}{\delta A_{\text{Neel}}(\omega)} \hat{\Delta} \right]_{\eta=0} / \left(-\frac{T \partial \eta}{\partial T} \right)_{\eta=0} \quad (\text{S16})$$

where the sum over Matsubara and band indices is left implicit. Here we choose to consider the functional derivative with respect to $A(\omega)/\omega$ because at low frequencies this is the quantity which gives the scattering rate, while if A is concentrated at high frequencies the $1/\omega$ gives the usual BCS logarithm.

We see that the logarithmic derivative of the transition temperature (S16) is determined by the frequency dependence of the gap function (eigenvectors) $\bar{\Delta}_{i,n}$ – explicitly in the expectation value of $\delta \hat{K}_s$ and implicitly in the temperature

dependence of η . In figure S1, we plot the gap $\bar{\Delta}_{X,n}$ of the electron pocket for the cases $\xi_{\text{Neel}} = 0$ (red/solid line) and $\xi_{\text{Neel}} = 0.5a$ (blue/dashed line). The hole pocket gap displays a similar behavior. Clearly, the s-wave gap in the presence of Neel fluctuations decreases much faster as function of frequency than the gap of the “pure” s^{+-} state. Thus, the s^{+-} state adapts to the presence of (π, π) fluctuations by suppressing the gap at higher frequencies to avoid the main depairing effect, which comes from the high frequency components of the Neel spectrum.

IV. DEPENDENCE OF TRANSITION TEMPERATURE ON IMPURITY SCATTERING

The Bergmann-Rainer approach is also used to obtain the dependence of the transition temperature on impurity scattering:

$$\frac{\partial T_c^{\text{s-wave}}}{\partial \tau^{-1}} = \left[\hat{\Delta} \frac{\partial \hat{K}_s}{\partial \tau^{-1}} \hat{\Delta} \right]_{\eta=0} / \left(-\frac{\partial \eta}{\partial T} \right)_{\eta=0} \quad (\text{S17})$$

where τ^{-1} is the scattering rate due to impurity scattering. The reduced $\tilde{T}_c^{\text{s-wave}}$ is then obtained via the linear approximation $\tilde{T}_c^{\text{s-wave}} = T_c^{\text{s-wave}} + \tau^{-1} \left(\frac{\partial T_c^{\text{s-wave}}}{\partial \tau^{-1}} \right)$.

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